Derivative Analysis of Single Hop Dynamic Wireless Power Transmission System using Magnetic Resonance Principle

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Abstract—*This paper presents the derivative analysis of important parameters which is solely required to design an efficient Wireless Power Transmission System. The four-coil coupling system is used to determine the relation between the parameters like coupling coefficient, mutual inductance, resonant frequency, Transmitter and Receiver Capacitance between Transmitter and Receiver Coil or device. The important part of this paper is that, it brings out a derivative analysis for dynamic or mobile Transmitter and Receiver modules that supports for an effective and possible long distance Wireless Power Transmission System. This concept completely deals with the resonant frequency that lies in ISM Band (i.e) 2.46GHz. The result will include the graphical representation for the following relations,*

a. *Relation between coupling coefficient and mutual inductance of the two-coil system.*
b. *Relation between coupling coefficient and resonant frequency*
c. *Relation between distance of separation and resonant frequency required to maintain high coupling coefficient*
d. *For capacitive switching concept, Transmittance and Receiver Capacitance are determined for various distance at a constant resonant frequency.*

Keyword*: Wireless Power Transmission System, Coupling Coefficient, Mutual inductance, Resonant frequency, ISM Band, Four Coil System.*

1. INTRODUCTION

The continuous market expansion of wireless products has encouraged the development of various wireless power transfer techniques into massive applications. Nowadays, wireless power transfer technology are used in many applications such as implantable medical devices **[1]– [3]**, electric vehicle charging **[4]– [6]**, and portable consumer electronic devices **[7], [8]**. The inductive power transmission technology with a resonance frequency at several kilohertz has been widely adopted for transferring low powers (a few watts) across short-range distances (several millimeters) **[9]– [12]**. To extend its power transfer capability to a higher power range (tens of watts) and distance range (e.g., several centimeters), the wireless power transmission (WPT) system operating at the range of megahertz has recently been developed **[3], [13], [14]**.

With the development and application of wireless power transmission technology based on magnetic resonance coupling, parameters optimal design and optimal control for the power transmission process have become the focus of research **[15]**. The Magnetic Resonance has significant advantage in transmission distance compared with electro-magnetic induction, this technology has intrinsic limitation as the load absorption power is sensitive to variations in the operating parameters, and small differences in operating and resonance frequency will reduce transmission performance significantly.

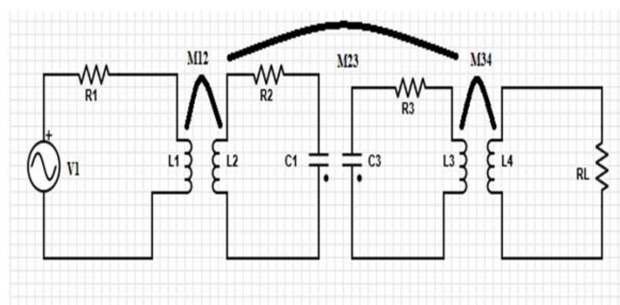

Fig. 1: Equivalent Circuit Model of a Resonating System

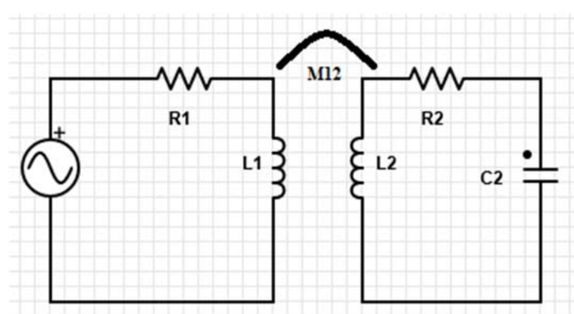

Fig. 2: Equivalent Circuit of a Two coil system

2. CIRCUIT MODEL

The equivalent circuit model of this resonant system, based on mutual inductance theory, is shown in **Fig. 1 [16]**. There, $L1$, $L2$, $L3$, and $L4$ are the self-inductance of the driving, transmission, receiving, and pick-up coils, C2 and C3 the respective resonant capacitance of transmitter and receiver coils, R2 and R3 are the respective resistances of the transmitter and receiver coils., $R1$ the sum of the power amplifier output resistance and the equivalent resistance of the driving coil, RL the equivalent load of the system including the resistance of the pick-up coil, $V\dot{}_1$ the excitation voltage source, and Mij and kij the respective mutual inductance and coupling coefficient of any pair of coils, with $Mij = k\ ij\sqrt{LiLj}$ and $0 \le k\ ij \le 1$. As shown in the **Fig. 1**, the four-coil system is the combination of two double coil system. So, initially the two-coil system can be analyzed for determining the parameters like coupling coefficient, mutual inductance, impedance of the coil. The two-coil system is shown in **Fig. 2**. The double coil system can be analyzed using image impedance method.

Initially, this double coil system is used to determine the relation between M_{12} and impedance. Finally, this concept will prove the relation between the capacitance of either Transmitter or receiver coil with coupling coefficient.

Fig. 3 is the T- circuit model for inductance part of **Fig. 2** is shown below, which is used to determine the mutual inductance parameter $\mathbf{M_{12}}$.

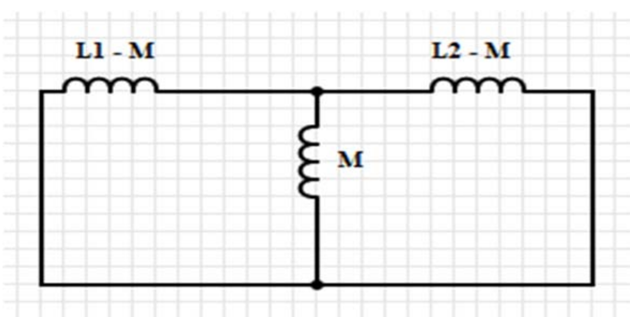

Fig. 3: Equivalent Circuit of a Two coil system

The relation between impedence and mutual inductance M_{12} is determined below,

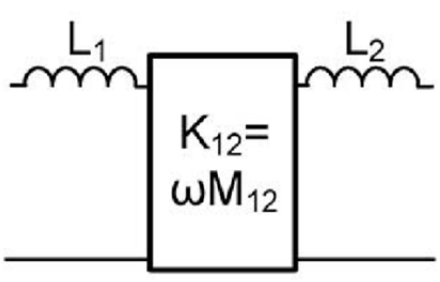

Fig. 4: Coupling coefficient equivalent circuit for Mutual Inductance

In the below derivation, the Mutual inductance $\mathbf{M_{12}}$ is denoted as **M** for simplicity and the resonant frequency is mentioned as **w**. Thus, the below derivaion results in the relation between the mutual inductance and impedence of either Transmitter or Receiver Coil.

$\mathbf{Z_a} = [jw\,(L_2\text{-}M) + (1/jwC_2) + R_2]$ **1**

$\mathbf{Z_b} = Z_a \parallel jwM$

Therefore,

$\mathbf{Z_b} = [jw\,(L_2\text{-}M) + (1/jwC_2) + R_2] \parallel jwM$ **2**

$\mathbf{Z_b} = (1/jwC1) + jw(L_1\text{-}M) + [\{jw^3C_2(M\text{-}L_2) + jwM - w^2MC_2R_2\}/\{w^2C_2(M\text{-}L_2) + 1 + jwR_2C_2 - w^2MC_2\}]$ >>>> **3**

If resonant frequency of Tx and Rx coils are same, then

$L_1C_1 = L_2C_2$ **4**

Substitute **4** in **3.** After simplification, following relation is obtained.

$Z_a = (w\,M_{12})^2 / Z_b$ **5**

Where,

Z_a = input impedance or Transmitter impedance

Z_b = output impedance or Receiver impedance w = Resonant Frequency M_{12} = Mutual inductance between Transmitter and Receiver Coil.

Since **Fig. 3** is redrawn as **Fig. 4**, The series impedance becomes zero. From **5**,

Assume, $Z_a = R_g$; $Z_b = R_l$;

$R_g * R_l = (w * M_{12})^2$ **6**

Substitute **4** in **6**

$\mathbf{M_{12}} = (R_g * R_l * L_2 * C_2)^{1/2}$ **7**

Where, the M_{12} is mutual inductance between Transmitter and Receiver Coil.

The coupling coefficient between Transmitter and Receiver Coil is denoted as below,

$\mathbf{K_{12}} = M_{12} / [L_1 * L_2]^{1/2}$ **8**

Substitute **7** in **8**, Further simplify to get the below relation,

$\mathbf{K_{12}} = \{[R_g * R_l * C_2]/L_1\}^{1/2}$ **9**

Similarly, $\mathbf{M_{34}} = (R_g * R_l * L_4 * C_4)^{1/2}$ **10**

From the Fig. 1, the Mutual inductance like M_{12} and M_{34} is determined. This is observed in the following derivation, From the **Fig. 1**, the voltage across the mutual inductance M_{12} is denoted as V_2.

$V_2 = \{[jw * M_{12}]/[R_1 + jw * L_1]\} * V_1$ **11**

The **Fig. 5** is provided below, which denotes the double coil equivalent circuit of **Fig. 1,**

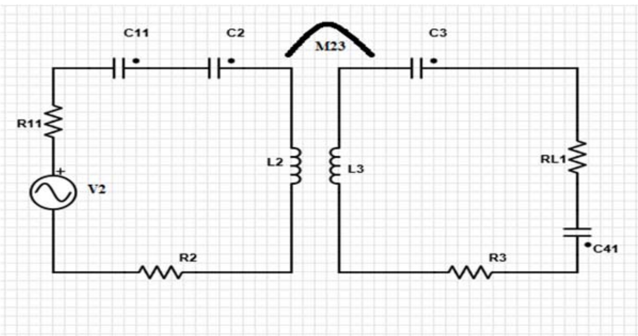

Fig. 5: Equivalent Circuit of a Two Coil System

From the above figure, consider the reonant frequency are same.

The input impedence Z_1 is determined below,

$\mathbf{Z_1} = [w * M_{12}]^2 \ / [R_1 + jwL_1] = [R'_1 + \{1/jwC'_1\}]$ **12**

By solving **12 ,** we can obtain the equaiton for R'_1 and C'_1,

Where,

$\mathbf{R'_1} = [R_1*(w * M_{12})^2]/[R_1^2 + (w*L_1)^2]$ **13**

$\mathbf{C'_1} = [R_1^2 + (w*L_1)^2]/[w^2 * L_1 * (w*M_{12})^2]$ **14**

The output impedance Z_{out} is determined below,

$Z_{out} = (w*M_{34})^2/(R_3 + jw*L_3) = R'_3 + (1/jw*C'_3)$ **15**

By solving **15 ,** we can obtain the equation for R'_3 and C'_3,

$\mathbf{R'_3} = [R_3 * (w*M_{34})^2]/[w^2 * L_4 * (w*M_{34})^2]$ **16**

$\mathbf{C'_3} = [R_3^2 + (w * L_4)^2]/[w^2 * L_4 * (w * M_{34})^2]$ **17**

From the equation **13** and **14,** we can determine the equation for R_{22}.

$\mathbf{R_{22}} = R'_1 + R_2$ **18**

Substitute **13** in **18,** and simplify to get the below equation.

$\mathbf{R_{22}} = [2 * R_1^2 * R_2 + w^2 * L_1^2 * R_2] / [R_1^2 + w^2 * L_1^2]$ **19**

From the equation **16** and **17,** we can determine the equation for R_{33}.

$\mathbf{R_{33}} = R'_3 + R_4$ **20**

Substitute **16** in **19**, simplify to get the below equation.

$\mathbf{R_{33}} = [2 * R_3^2 * R_4 + w^2 * L_3^2 * R_4] / [R_3^2 + w^2 * L_3^2]$ **21**

From [17], the coupling coefficient K_{23} of **Fig. 1** is denoted as follows,

$\mathbf{K_{23}} = [1/ (L_2 * L_3)^{1/2}] * \{[(R_{22}^2 + X_{22}^2) * (R_{33}^2 + X_{33}^2)^2] / [R_{33}^2 * w^4 + X_{33}^2 * w^4]\}$ **22**

From [17], At resonant condition, $X_{22} = X_{33} = 0$;

Therefore, the coupling coefficient can be denoted as follows,

$\mathbf{K_{23}} = \ [R_{22} * R_{33}]^2 / [\ (L_2 * L_3)^{1/2} * w^4\ ]$ **23**

Assume, the coupling coefficient is 100% (i.e.) $K_{23} = 1$ for equation **23.**

We can find the relation between distance vs resonating frequency, Transmitter Capacitance and Receiver Capacitance. This can be obtained with the help of the following derivation.

From **23,** the resonating frequency is denoted as follows,

$\mathbf{w} = \{[R_{22} * R_{33}] / [L_2 * L_3]\}^{1/2}$ **24**

The equation **6** can be written as follows,

$\mathbf{M} = d * (2 * pi * c) * (R_g * R_l)^{1/2}$ **25**

Where,

M = mutual inductance between Transmitter and Receiver Coils,

c = Velocity of Light,

d = Distance of Separation between Transmitter and Receiver Coils.

Substitute **25** in **8,** we get the following equation,

$\mathbf{K_{23}} = [\{d * 2 * pi * c\} / L_3] * [\{R_2 * R_3 * C_2\}/C_3]^{1/2}$ **26**

From equation **26 ,** we can determine L_3.

$\mathbf{L_3} = [d * 2 * pi * c] * [\{R_2 * C_2 * R_3\} / C_3]^{1/2}$ **27**

Substitute equation **27** in **24**, we can obtain the following equation.

$\mathbf{w} = [\{[R_{22} * R_{33} * (C_3)^{1/2}]\} / \{L_2 * d * 2 * pi * c * (R_2 * R_3 * C_2)^{1/2}\}]^{1/2}$ **28**

From equation **28,** we will be able to determine the relation between distance vs resonating frequency, Transmitter and Receiver Capacitance.

3. GRAPHICAL REPRESENTATION

This section presents the graphical representation of the relation between the distance vs resonant frequency, Transmitter and Receiver Capacitance, Coupling coefficient vs resonant frequency.

4. RELATION BETWEEN COUPLING COEFFICIENT AND RESONANT FREQUENCY

The resonant frequency is the frequency at which the Transmitter and Receiver coils gets coupled with each The resonant frequency is varied from 1.74 MHz to 20 MHz and its counterpart optimal coupling coefficient is analyzed. The graphical representation of this relation is shown in **Fig. 6.**

5. RELATION BETWEEN DISTANCE OF SEPARATION AND RESONANT FREQUENCY

The distance of separation is the distance between the Transmitter and Receiver coil which is denoted as **d.** The graphical representation of this relation can be obtained using equation **28.** The main objective in studying about this relation is, in real time application the transmitter and receiver coils may be in static or in dynamic mode but this is operated under a frequency called resonant frequency. The **Fig. 7** denotes the graphical representation between the distance of separation and Resonant frequency. The resonant frequency is calculated for different distance values which is ranging from 0 to 32 meters.

6. RELATION BETWEEN TRANSMITTER CAPACITANCE AND DISTANCE OF SEPARATION

The four-coil system (as shown in **Fig. 1**) has two capacitances namely sender and receiver capacitance. This section shows the graphical representation for the relation between transmitter capacitance vs distance of separation between coils for different values of receiver capacitance. The **Fig. 8** denotes the graphical representation for the above-mentioned relation.

7. CONCLUSION

By improving the system resonance frequency or coupling coefficient $k12$, $k34$, the power transmission stability can be improved while power transmission performance sensitivity to variations in operating parameters can be decreased. **The figures 6,7 and 8 are attached after the reference section.**

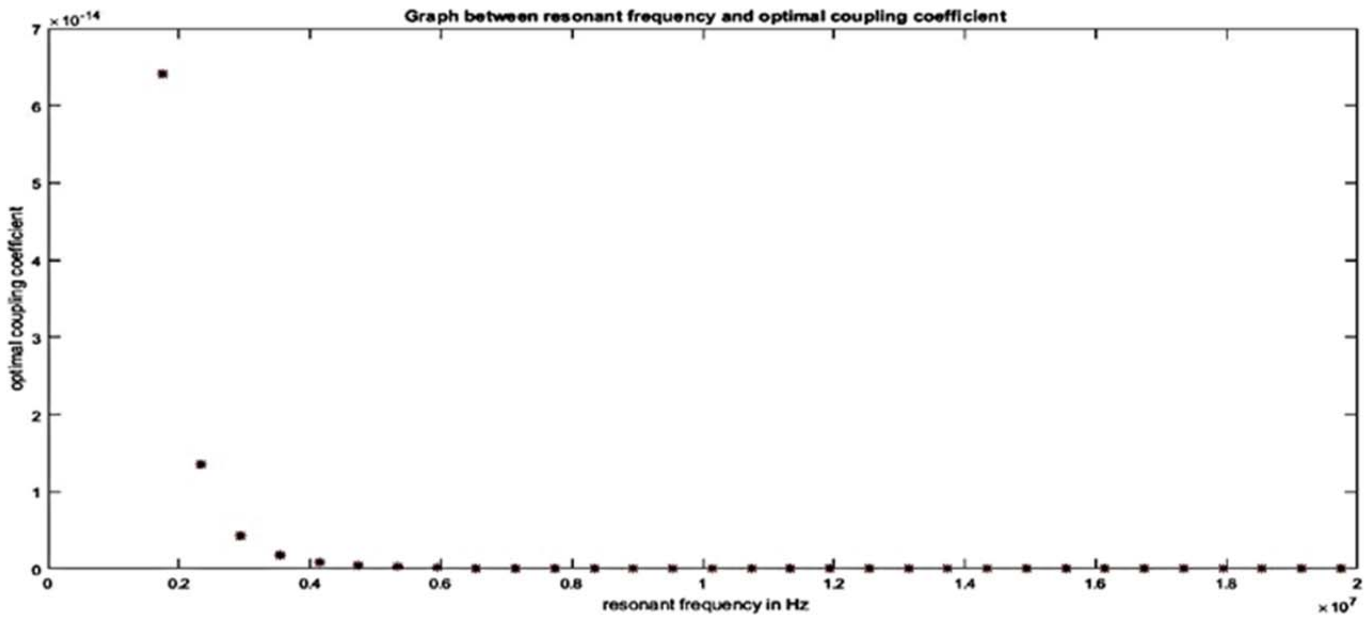

Figure 6. Relation between Resonant frequency and optimal coupling coefficient

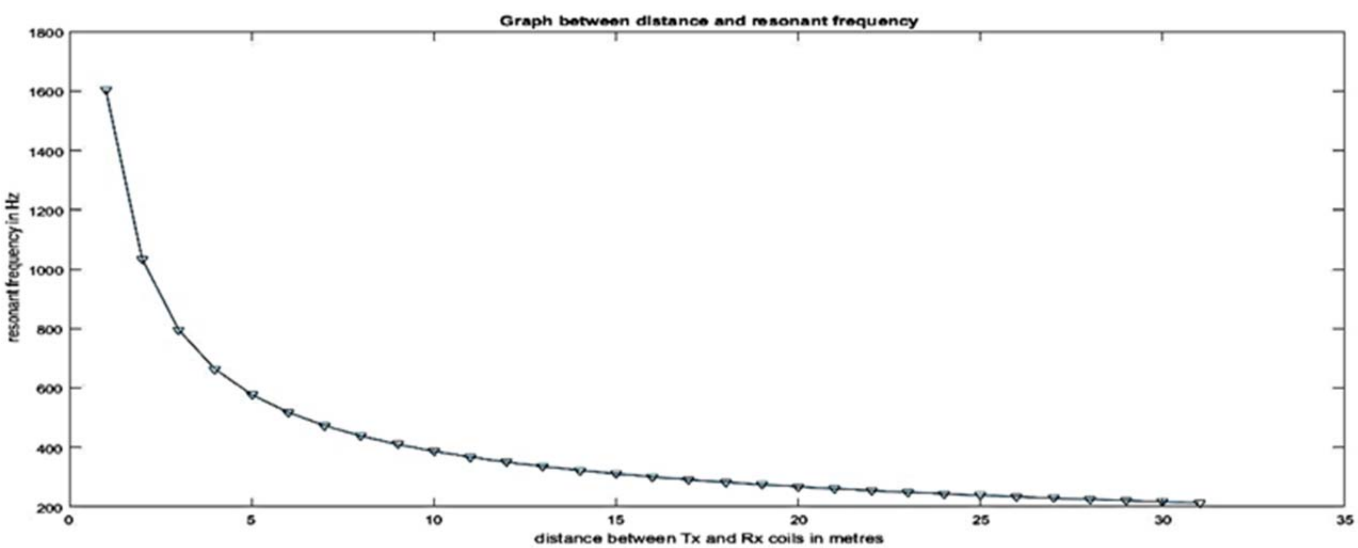

Figure 7. Relation between distance of seperation and Resonant frequency in Hz

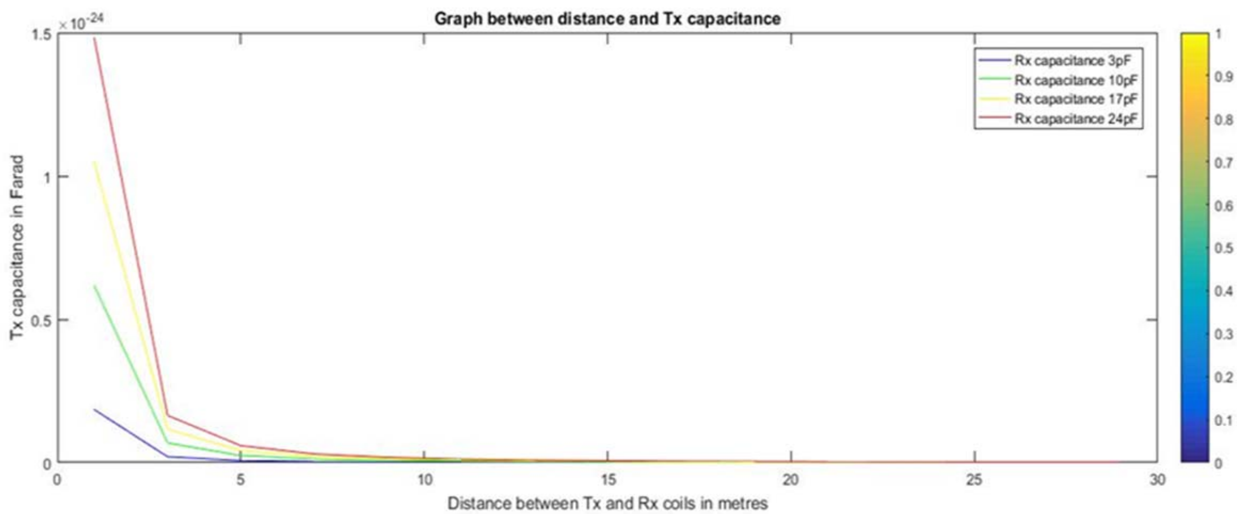

Figure 8. Relation between Distance of Separation and Transmitter Capacitance

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